PERT Charts

The list of essentials of a project plan includes an activity/event network plan. This is a pictorial representation of how an event or activity progresses, step-by-step, at each decision level. To go a step further, add time to an activity/event chart and come up with a time/activity/event chart.

There are several types of time/activity/event charts. The most frequently used in project planning and control are the Gantt and PERT charts.

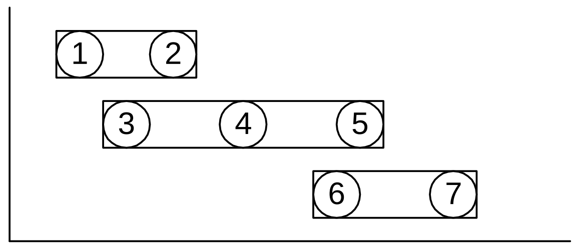
Background

PERT stands for Project Evaluation and Review Technique. It was developed for the United States Navy as a mechanism to keep track of large projects by Booz, Allen and Hamilton (management Consultants).

It is a pictorial method of minimizing production delays, interruptions and conflicts. It allows coordination and synchronization of various parts of a job. The work is controlled and orderly.

Process

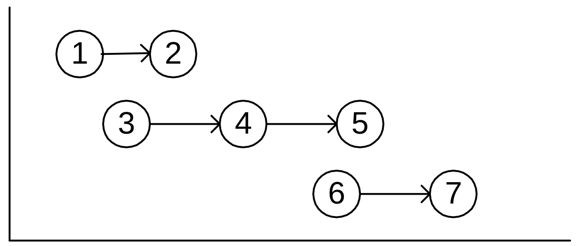
Start with a Gantt chart.



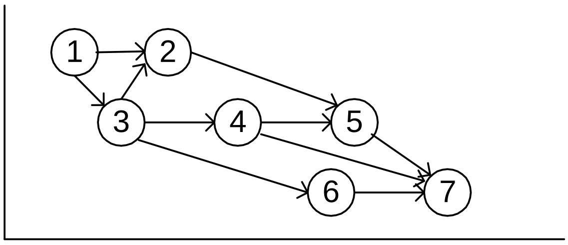
This chart does not clearly show the interrelationships between the numeric milestones. They can only be inferred from the chart.

There are three transition steps from this Gantt chart to a PERT chart:

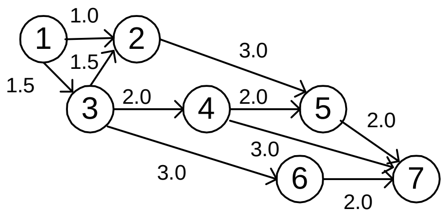
1. Remove the rectangles. It still does not show the interrelationships.



2. Add the relationships using lines with arrows to show the direction of work flow.



3. Omit the axis lines, add time values to the arrows to indicate time to move from milestone to milestone.



Advantages

- PERT chart indicates interrelationships between milestones.
- It makes redundant the designation of "task" because the project is now an integrated whole.
- The value of each leg is shown.
- You can network very complex projects.
- You can use probability theory to obtain time values even when uncertain.

NOTE: The finished PERT chart (or PERT diagram) should be easy to read. The length of the arrows (activities) is NOT representative of the time value of the activity, but rather is determined by the most effective (readable) layout.

Fundamentals

Event – a specific accomplishment that occurs at a recognized point in time (also known as a milestone)

Activity – work required to complete the event.

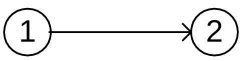
Always – two events, one activity.

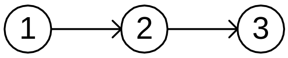
NOTE: Activities, not events require time and resources.

Activity flows left to right, top to bottom.

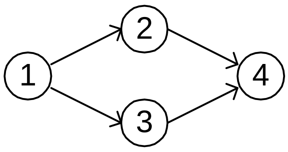
Network – several events and activities combined into a drawing

Here are some examples:

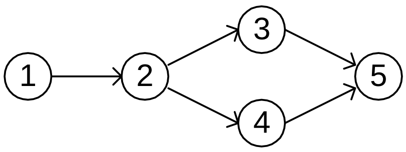
Network	Event	Activity
	1 2	1-2



1	1-2
2	2-3
3	



1	1-2
2	1-3
3	3-4
4	2-4



1	1-2
2	2-3
3	2-4
4	3-4
5	4-5

Key Concepts

A. Pert is time based. The calendar week is the basic unit, written to 1 decimal:

Number of days to complete / number of days in a work week = n.n

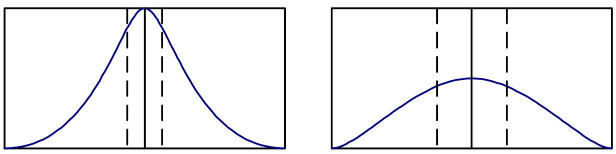
Hours are not used because the level of detail for would be too high (i.e. if something is 1 hour late over a 4 week period, that is less than 1%).

B. There is chance in all estimates. Notice how these "time estimate" terms all contain measures of chance (probability):

Best Guess   Worst Guess   Pessimistic   Optimistic   Realistic

C. Probability – necessary to understand how PERT charts can provide significant control over a project.

Two types of bell curve:



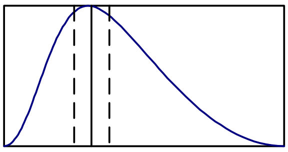
In the first bell curve, the values are tightly grouped about the mean. In the second curve, the values are widely dispersed away from the mean. Which is correct? (answer: Both. It depends on the data). However, you can make meaningful statements about the data in each curve. The Standard Deviation is the mathematical description of the difference between the two curves, above. It is a measure of the scatter of the data.

Standard Deviation (σ) = $\sqrt{(\sum(i - X)^2 / n)}$

Where i is the individual data point
X is the Mean for the data set
n is the number of points in the data set

About 68% of all values are within +/- one standard deviation from the mean.
About 95% of all values are within +/- two standard deviations from the mean.
Greater than 99% of all values are within +/- three standard deviations from the mean.

If the bell curve is symmetrical (both sides the same size and shape), then it is called a Gaussian distribution. If the bell curve is not symmetrical, it is called a Beta distribution. Beta distributions are actually more common in real life data than Gaussian distributions.



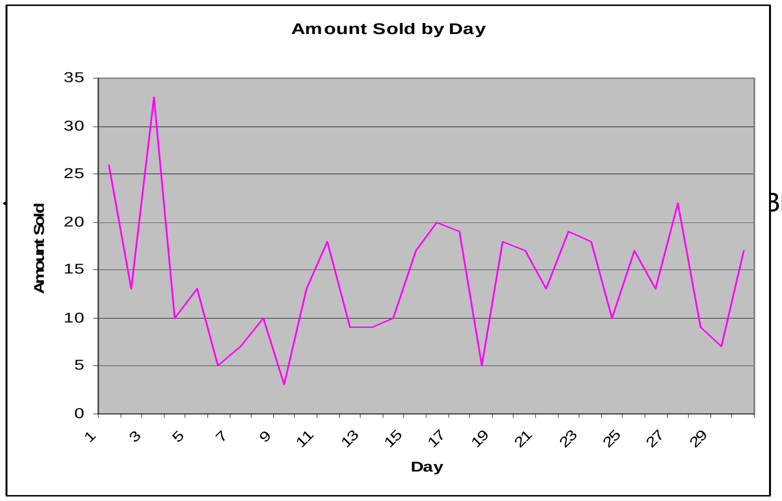
Example:

Day	Amount Sold	I – MEAN	(I-MEAN)**2
1	26	12	144
2	13	-1	1
3	33	19	361
4	10	-4	16
5	13	-1	1
6	5	-9	81
7	7	-7	49
8	10	-4	16
9	3	-11	121
10	13	-1	1
11	18	4	16
12	9	-5	25
13	9	-5	25
14	10	-4	16
15	17	3	9
16	20	6	36
17	19	5	25
18	5	-9	81
19	18	4	16
20	17	3	9
21	13	-1	1
22	19	5	25
23	18	4	16
24	10	-4	16
25	17	3	9
26	13	-1	1
27	22	8	64
28	9	-5	25
29	7	-7	49

30	17		3	9
TOTAL	**420**	**SUM**		**1264**

MEAN = Amount Sold / Total Days = 420 / 30
MEAN 14
Standard Deviation = SQRT((SUM(I - MEAN)2)/NUM) = 1264/30**
S.D. 6.491019

A. Plot Amount Sold vs. Days



BE REMOVED WHEN COMPLETE.>

B. Plot Amount Sold vs. Number of Days That Amount Was Sold

Amount Sold (sorted)	Amount Sold	Number of days this amount was sold
33	33	1
26	26	1
22	22	1
20	20	1
19	19	2
19	18	3
18	17	4
18	13	5
18	10	4
17	9	3
17	7	2
17	5	2
17	3	1
13		

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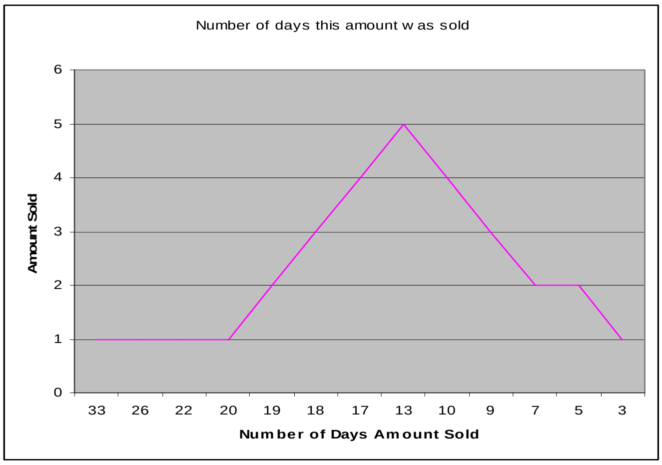


This is a Beta Distribution (not very smooth, but a beta distribution nontheless).

In PERT chart usage, the values to the left of the mean are classified as OPTIMISTIC, and those to the right of the mean are PESSIMISTIC. The values within 1 Standard Deviation of the mean are classified as MOST LIKELY (let's label them "m". MOST OPTIMISTIC values (let's label them "a") are those greater than 3 SD from the mean to the left and MOST PESSIMISTIC values (let's label them "b") are those greater than 3 SD from the mean to the right.

The EXPECTED TIME of a PERT chart event is a weighted average of these three values:

te = (a + 4m + b) / 6

Note how the weighting gives the MOST LIKELY value four times the probability of the least likely events.

So, for our example above, most likely sold is 14, most optimistic is 33 and most pessimistic is 3. The expected number sold is (33 + 4 * 14 + 3)/6 or 45.33, or 45.

NOTE ON PRECISION. Precision in the answer is more than just how many decimals a calculator can report. Precision must have meaning. In this example, the data is "Amount sold". This is an integer quantity, as you cannot sell 1/2 of a unit (at least without knowing more about the data, and our problem is silent on this count – so take the simplest solution). Therefore, 45.33 does not have meaning, while 45 does.

Network Principles

1. Combining Networks.

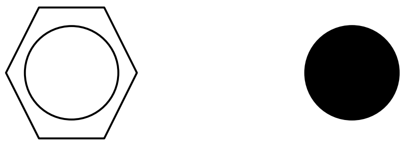
Given two networks, can we combine them into a single network? Yes, if one network represents a sub-system of another network, AND if there is a common event to both networks. The two can then be combined into a single, larger network.

For example, consider the manufacture of an aircraft. You may have one PERT network for airframe assembly, and another for engine assembly. Suppose there is an event in both networks called "mate engine to airframe". This event can be used as an interface event to combine both networks into a single larger network.

If there is no common event but you know the networks are related, a common event can be created to allow the networks to be merged.

2. Interface Events.

Interface events are events common to more than one network. They may be used to combine the networks (above), or the networks may be kept separate. However, interface events signify events where more than one network must come together before they can proceed. Consequently, even if the networks are not combined (perhaps for management reasons), they event must be marked as special so that the effect of the other network can be calculated. The symbol(s) for an Interface event is shown below.



3. Zero Time Events.

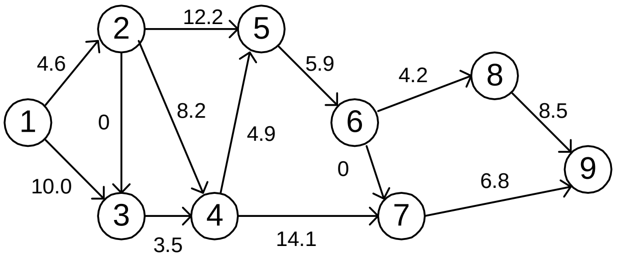
Some events are necessary to indicate important milestones in a project. However, these events may not take any time to complete, but still must be indicated. Zero Time Events are shown with a time of "0" on the activity.

For example, after producing a report, it may be deemed essential to show "deliver report to vice president" on the PERT chart. However, this activity may be considered "done" as soon as the report has been proofread (i.e. it is emailed to all required parties). Thus, this event is shown, but as a zero time event.

4. Earliest Expected Date (TE)

This is the earliest date the project can possibly be completed. It is calculated by adding up the optimistic times of all possible paths through the network, and reporting the minimum time.

NOTE: this calculation is made much easier once we discover the Critical Path (item #6).



1-2	1-3	2-3	2-4	2-5	3-4	4-5	5-6	6-7	4-7	6-8	8-9	7-9		
4.6	10.0	0.0	8.2	12.2	3.5	4.9	5.9	0.0	14.1	4.2	8.5	6.8		
4.6				12.2			5.9			4.2	8.5		35.4	1-2-5-6-8-9
4.6				12.2			5.9	0.0				6.8	29.5	1-2-5-6-7-9
4.6			8.2			4.9	5.9			4.2	8.5		36.3	1-2-4-5-6-8-9
4.6			8.2			4.9	5.9	0.0				6.8	30.4	1-2-4-5-6-7-9
4.6			8.2						14.1			6.8	33.7	1-2-4-7-9
4.6		0.0			3.5	4.9	5.9			4.2	8.5		31.6	1-2-3-4-5-6-8-9
4.6		0.0			3.5	4.9	5.9	0.0				6.8	25.7	1-2-3-4-5-6-7-9
4.6		0.0			3.5				14.1			6.8	29.0	1-2-3-4-7-9
	10.0				**3.5**	**4.9**	**5.9**			**4.2**	**8.5**		**37.0**	**1-3-4-5-6-8-9**
	10.0				3.5	4.9	5.9	0.0				6.8	31.1	1-3-4-5-6-7-9
	10.0				3.5				14.1			6.8	34.4	1-3-4-7-9

In the above example, the earliest expected date is **37.0** days.

To recap: the earliest expected date is the *shortest time* to complete the project for a given network diagram. It is **not** the shortest (smallest) summation of activity times, but rather the longest path through the network. You cannot complete the project any sooner than it takes to complete the longest path through the network.

5. The Effect of Zero Time Activities.

By definition, zero time activities have no impact. However, they still must be calculated into the overall time estimates, and may influence the critical path (see above example).

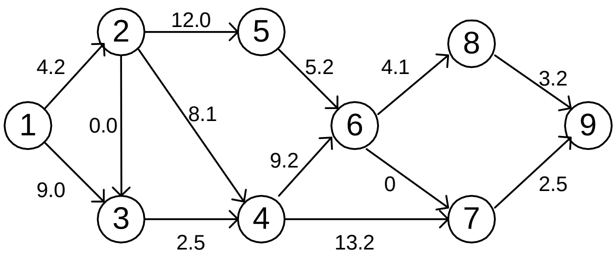
6. Critical Path

For every network, there exists a path which determines the earliest expected date. This is the CRITICAL PATH. By definition, any delay in any activity on the critical path will increase the earliest expected date (i.e. make the project late). In the example above, the critical path is the one that generates the earliest expected date – the path **1-3-4-5-6-8-9 (37.0)**

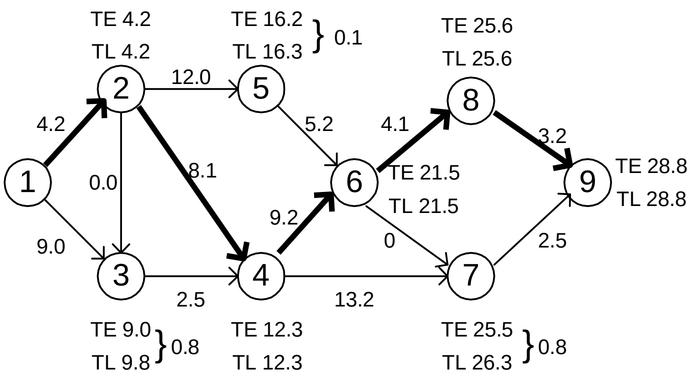
7. Latest Allowable Date (TL) and Slack Time (SL)

The Latest Allowable Date is the overall time you can be late on a non-critical activity and still not be late overall. By definition, TL must equal TE on the critical path. Where TL is greater then TE, there is slack time. The difference between TE and TL is called the Slack Time. NOTE: Although not very common, it *is* possible to have slack time on the critical path.

Example:



Solution:



Network Re-planning

Network re-planning is the act of adjusting the network to reduce the overall project time by moving labor to critical activities in order to minimize the slack time.

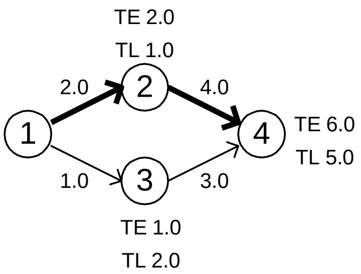
Procedure:

1. Make a slack time listing.
2. Adjust the labor
3. Modify the network (if necessary)

In order for this to be possible, the labor must be the same type i.e. all plumbers on a plumbing job. If the labor is different (i.e. electricians and plumbers), then you cannot engage in network re-planning unless you relax the technical specifications accordingly (this may cause other problems!).

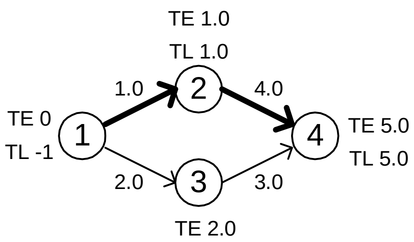
Also in this example, TL = 5.0 is not calculated from the network, but is an arbitrary figure imposed by some outside agency. That is, while the network shows TE=6 meaning that it will currently take 6 weeks to complete the project, an actual completion date (TL) of 5 weeks is required (contract, external time considerations, availability of labor or supplies, etc.). It is because of this arbitrary imbalance that network re-planning is necessary to complete the project “on time” (TL=5.0).

Example:



(Note: TL is not calculated from the network, but has been SPECIFIED as 5.0 in the problem)

Solution:



Critical Path Method vs. PERT

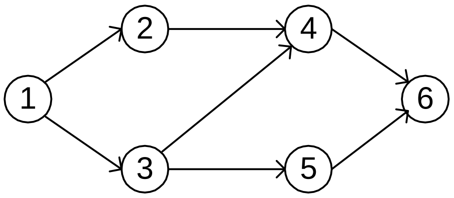
The Critical Path Method (CPM) is an enhancement of PERT chart use which can be very beneficial in large projects. However, it adds cost to the management of the project as you prepare multiple PERT charts.

In the CPM, you prepare two PERT time charts and two cost estimates. These are called "Normal" and "Crash". The normal PERT chart is as we have already discussed. The "Crash" PERT chart is one in which no cost is spared in order to reduce time to a minimum. The difference in cost is called the "Cost to Expedite" (CTE).

An analysis is then done to estimate the best plan of action based on the two estimates. A curve is drawn showing Cost vs. Time, and the values for crash and normal are plotted. A straight line is drawn between these two points. In practice, the line between crash and normal is not straight. Compared to an ideal straight line between normal and crash, if the actual curve is below the line, then a small decrease in time will have a small cost. On the other hand, if the actual curve is above the line, then a small decrease in time will have a large cost.

Advanced CPM analysis may also involve calculating the difference between "crash all" and "uncrash non critical" activities. This may provide significant cost savings while still retaining the speed of a crashed critical path.

Example:

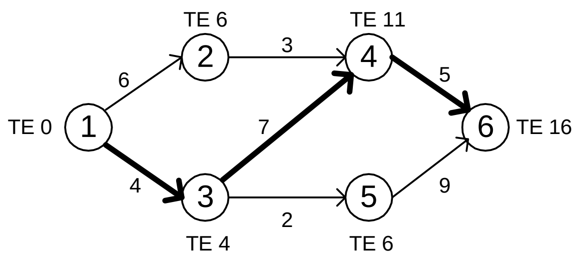


Activity	Normal Time	Crash Time	Normal Cost, $	Crash Cost, $	CTE
1-2	6	3	2000	6000	1333.333
1-3	4	2	3000	5000	1000
2-4	3	1	4000	5000	500
3-5	2	1	1000	3000	2000
4-6	5	3	4000	7000	1500
5-6	9	5	7000	10000	750
3-4	7	4	6000	10000	1333.333
TE	**16**	**9**	**27000**	**46000**	

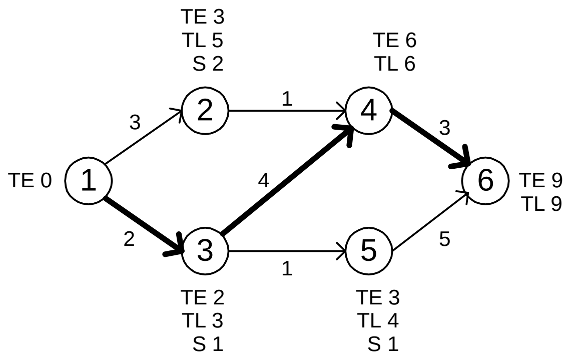
NOTE: CTE is calculated using the formula
CTE = Absolute Value [(Normal cost – Crash cost) / (Normal time – Crash time)]

The Normal and Crash data are then placed on the network and the critical path found:

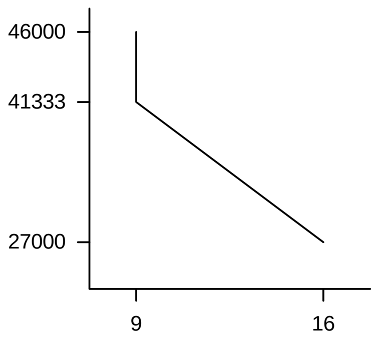
Normal:



Crash:



Finally, this is plotted on a Graph of Cost vs. Time


46000
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27000
9
16